An approach to the study of time, time-frequency and time-scale transformations for seismic migration problems

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Advisor
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Content

Antecedents

Research project ECOPETROL-COLCIENCIAS

Seismic pre-stack migration in depth by extrapolating wave fields using high performance computing for massive data in complex areas.

Cooperative research project: Universidad de Antioquia, Instituto Tecnológico Metropolitano -ITM, Universidad Industrial de Santander, Universidad de Pamplona.

Antecedents

Challenges in the oil industry

- Minimizing exploration costs.
- Minimize the degree of uncertainty in exploration.
- Improve subsurface characterization.
- Deepwater oil reservoirs.
- Deep reservoirs and complex areas.
- Small reservoirs in known areas.

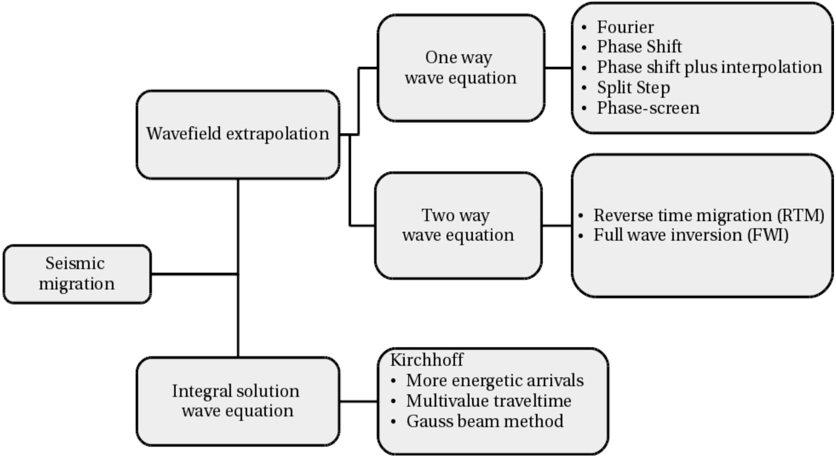
Antecedents

Marine seismic acquisition

https://youtu.be/ZesI8PevfAQ

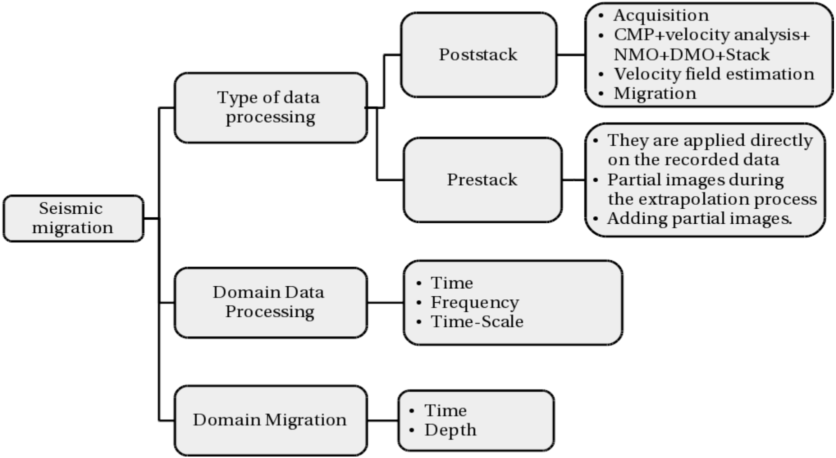
Antecedents

Seismic migration



Antecedents

Seismic migration



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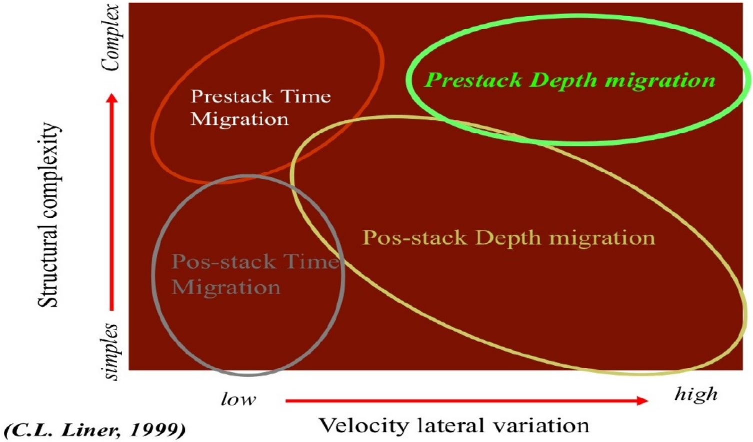

Background

- Geometric migration(until 1960)[1].

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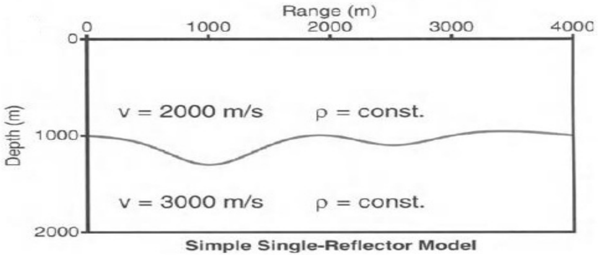

[1] Hagedoorn, 1954, [40]

Background

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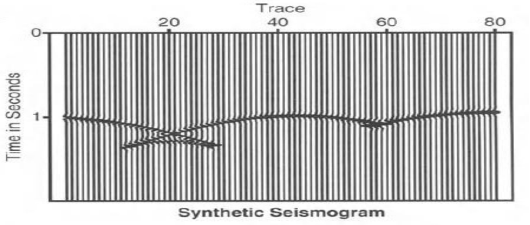

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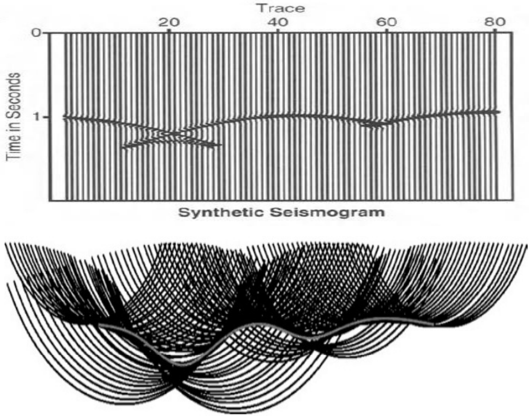

Background

- Geometric migration(until 1960)
- Diffraction summation migration (or diffraction stack)[2].

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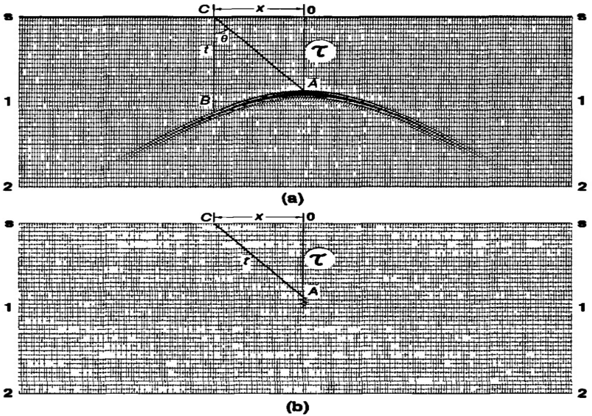
[2] Schneider, 1971, [52]

Background



Background

- Geometric migration(until 1960)
- Diffraction summation migration (or diffraction stack)
- Finite difference schemes for hyperbolic equations[3].

Claerbout, 1971, [13]
Schneider, 1978, [53]

Background

- Geometric migration(until 1960)
- Diffraction summation migration (or diffraction stack)
- Finite difference schemes for hyperbolic equations[3].
- Kirchhoff migration[4].

---

3 Claerbout, 1971, [13]
4 Schneider, 1978, [53]

Background

- Reverse time migration[5].

[5]McMechan, 1983, [47]
[6]Bleinstein, 1987, [4]

Background

- Reverse time migration[5].
- Kirchhoff migration enhanced the amplitudes and phases[6].

[5]McMechan, 1983, [47]
[6]Bleinstein, 1987, [4]

Background

- Migration by Fourier transform. (Migration in f-k domain)[7].

Stolt, 1978, [59]
Gazdag, 1978, [34]
Gazdag and Sguazzero, 1984, [35]
Stoffa et al, 1990, [58]

Background

- Migration by Fourier transform. (Migration in f-k domain)[7].
- Phase shift migration[8].

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- Phase shift plus interpolation (PSPI migration)[9].

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- Migration by Fourier transform. (Migration in f-k domain)[7].
- Phase shift migration[8].
- Phase shift plus interpolation (PSPI migration)[9].
- Split step migration[10].

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[7] Stolt, 1978, [59]
[8] Gazdag, 1978, [34]
[9] Gazdag and Sguazzero, 1984, [35]
[10] Stoffa et al, 1990, [58]

Background

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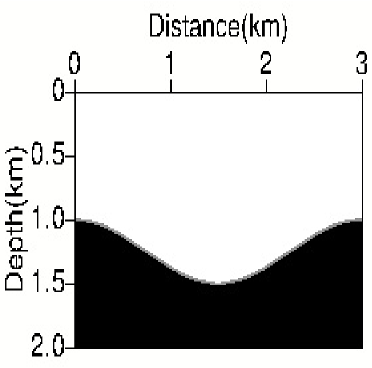
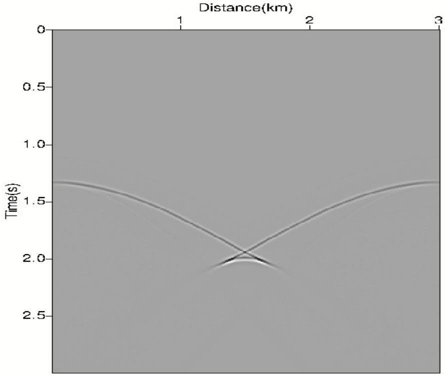
In recent years there have been extensions of these methods to three dimensions and pre-stack migration, with further refinements in terms of accuracy and efficiency.

Examples in migration

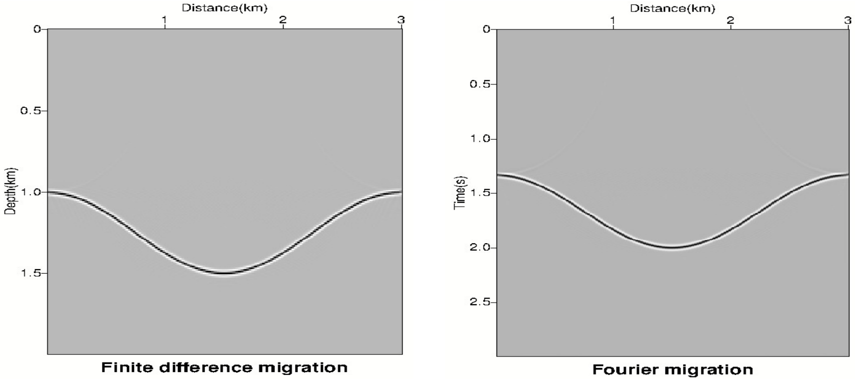
Sinclina model





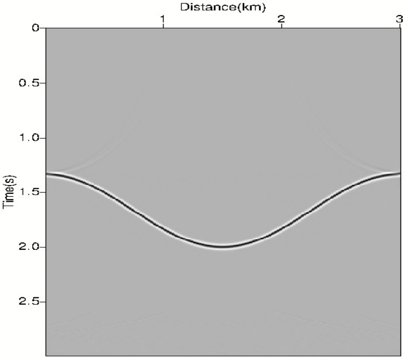
Examples in migration

Sinclina model

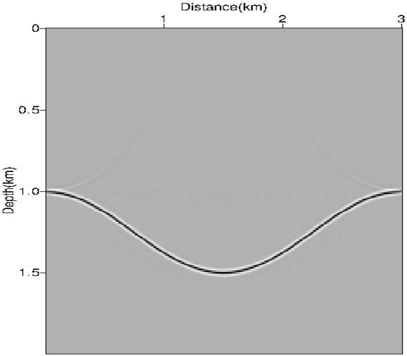


Examples in migration

Sinclina model



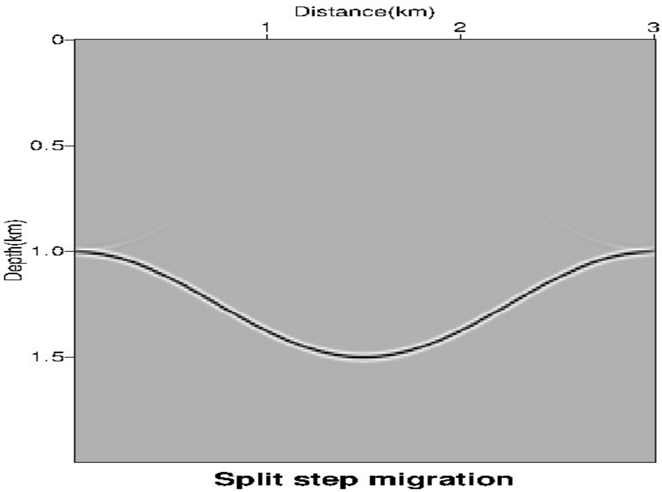
Phase shift migration



Phase shift plus interpolation migration

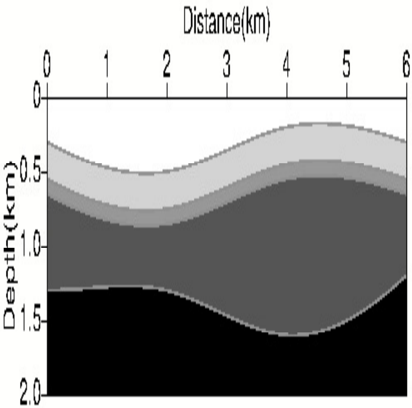
Examples in migration

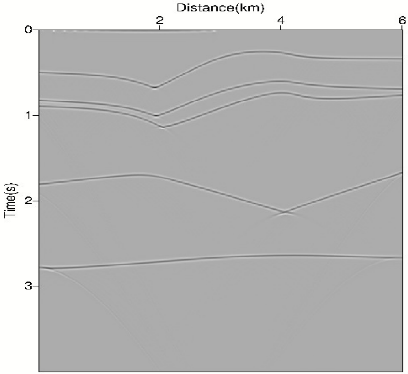
Sinclina model



Examples in migration

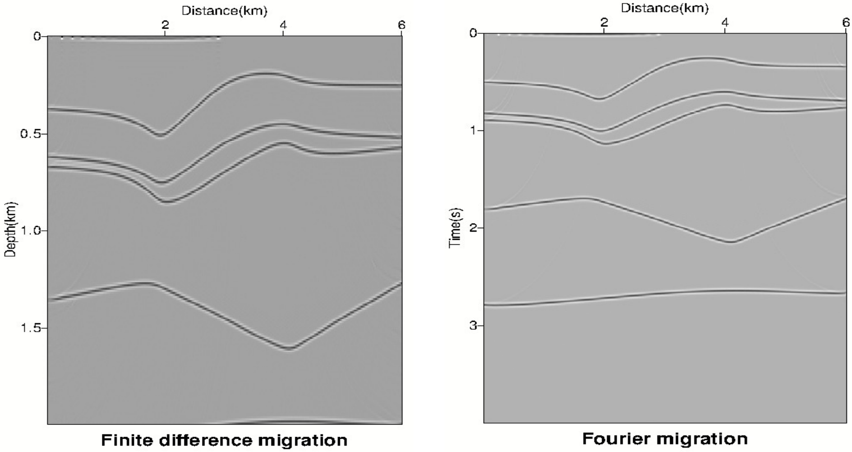
Five layer model





Examples in migration

Five layer model



Examples in migration

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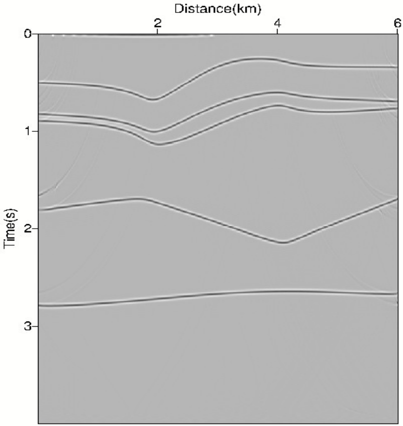
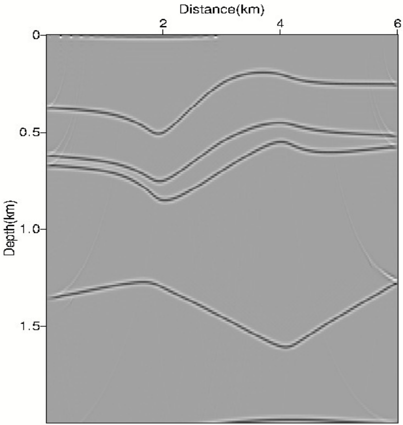


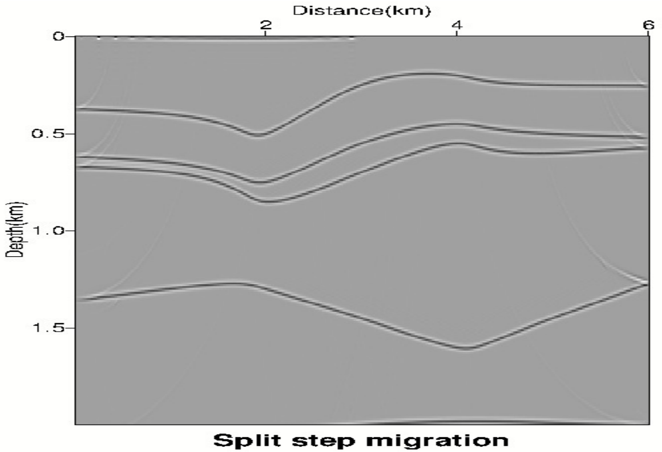
Phase shift migration



Phase shift plus interpolation migration

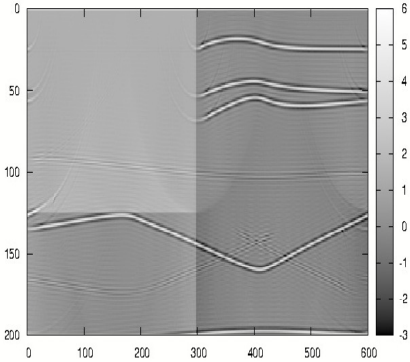
Examples in migration

Five layer model

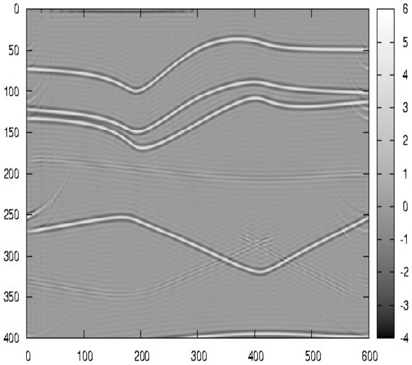


Examples in migration

Five layer model



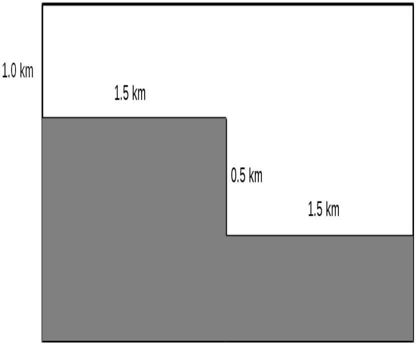
Phase shift migration $\Delta z = 0.01km$

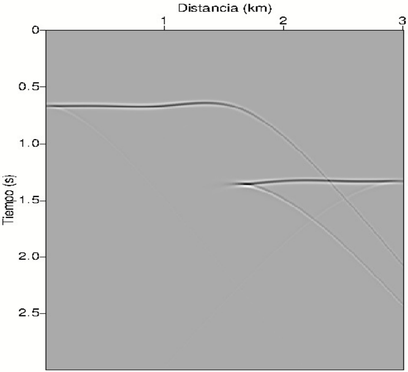


Phase shift migration $\Delta z = 0.005km$

Examples in migration

Step model





Examples in migration

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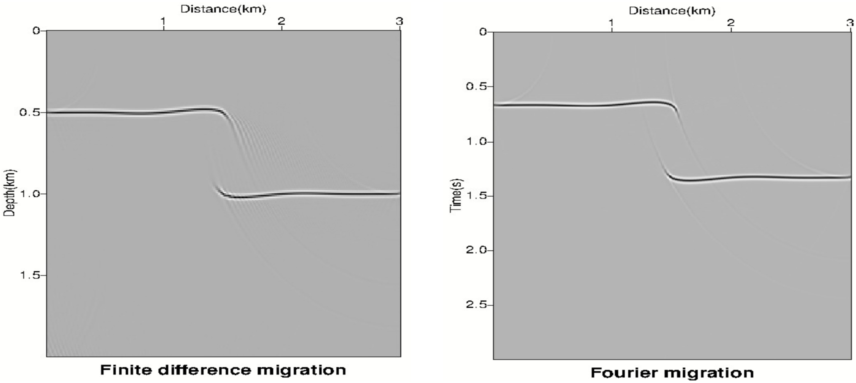

Examples in migration

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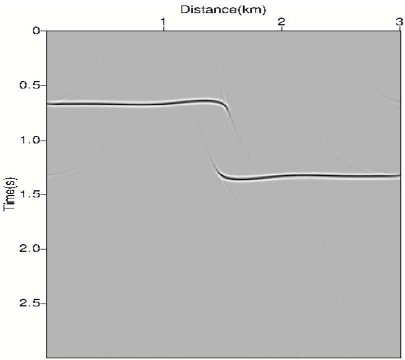
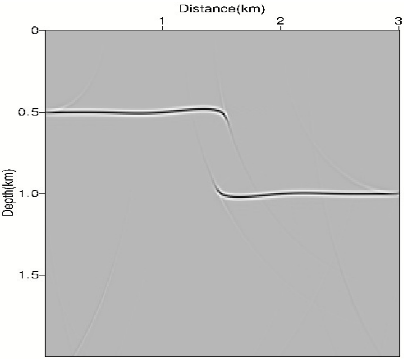


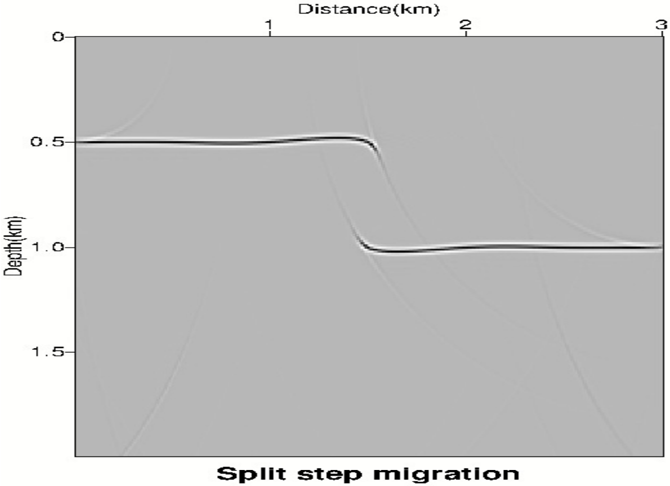
Phase shift migration



Phase shift plus interpolation migration

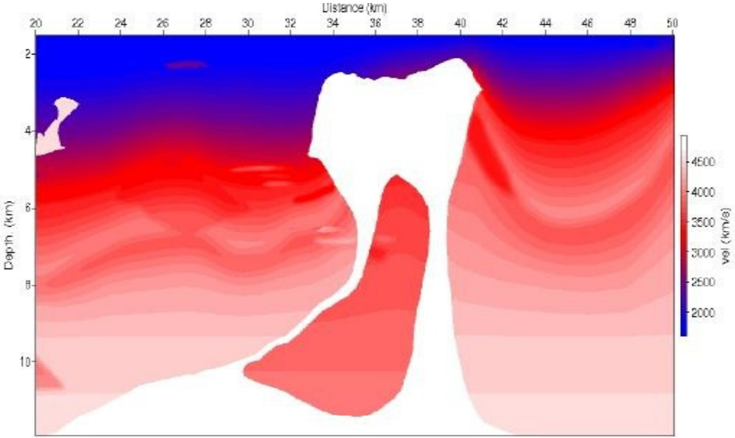
Examples in migration

Step model



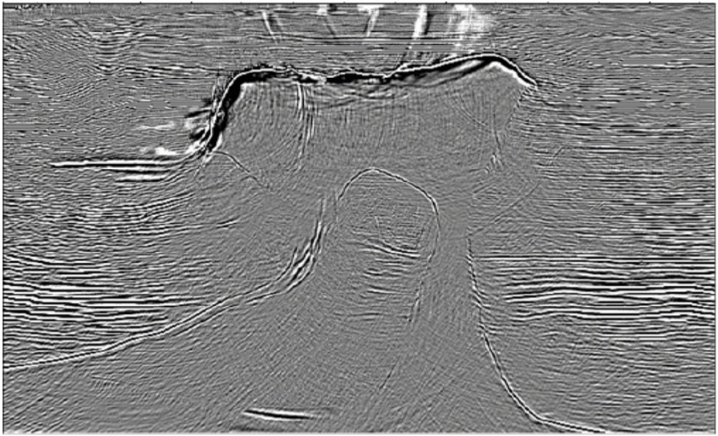
Examples in migration

BP model 2007



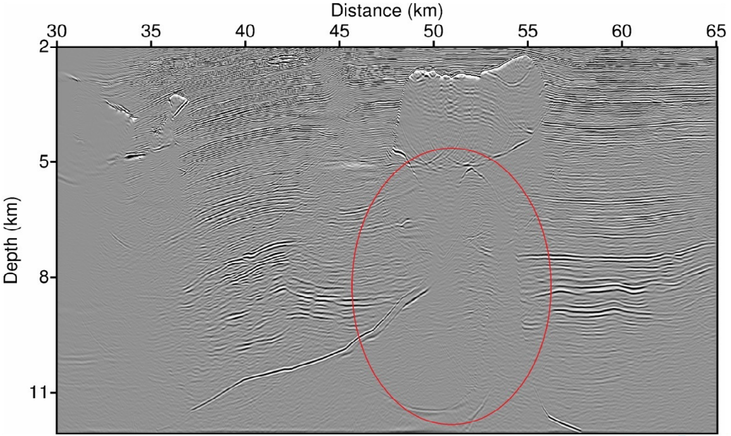
Examples in migration

Kirchhoff migration



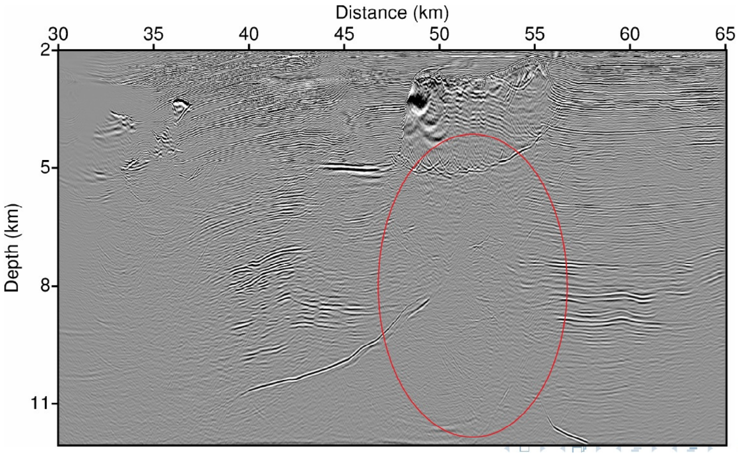
Examples in migration

PSPI migration



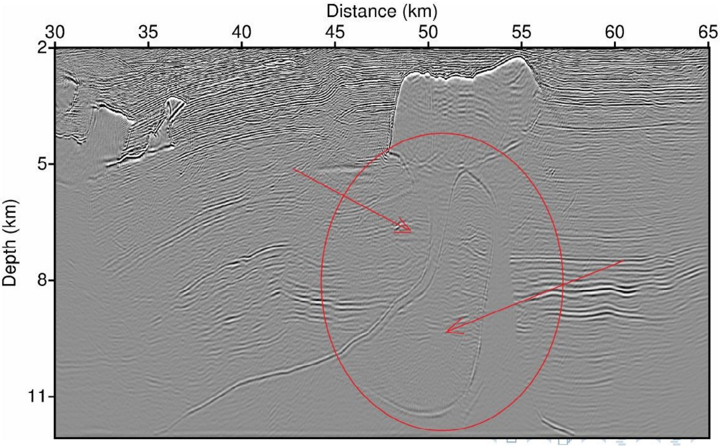
Examples in migration

Split-Step migration



Examples in migration

RTM migration



Background

Wavelet analysis

Methods for the analysis in the frequency-time domain (scale). Analysis of non-stationary signals or strong changes in small intervals. They are used in:

- Geophysics (Chakraborty, 1995, [9], Foufoula-Georgiou y Kumar, 1995, [28])
- Astrophysics (Starck, 2010, [56])
- Biology (Meyers,1993, [48])
- Signal and image in Medicine (Burt, 1989, [7], Polikar, 1997, [51])

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Wavelet analysis

- Compression fingerprints and images (Bradley, 1993, [6])
- Satellite images (Demirel, 2010, [16])
- Atmospheric analysis and turbulence (Gamage y Blumen, 1993, [31], Weng y Lau, 1994, [62], Farge, 1992, [26])
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Background

A brief history

- Gabor transform,1946[11].
- Gabor transform modified with dilated Windows[12].
- Morlet, (1982).Morlet wavelet basis[13].
- Goupillaud (1984) [14].

Gabor, 1946, [30]
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Background

A brief history

- Ortogonal wavelet transform[15]and pyramid algorithm[16].
- Seismic data compression[17]and satellite transmission [18].
- Emergence of new orthogonal wavelet transforms[19].

Daubechies, 1988,[15]
Mallat, 1989, [45], [46]
Luo et al,1992,[44], Bosman,1993,[5]
Donoho,1995,[19], Stigant,1995,[57]
Candes, 2006, [8],Herrmann, 2007, [42]

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Background

Wavelets in Meteorology and Oceanography

- Meyers et al [20].
- Szilagyi et al[21].
- Chen et al, 1987 [22].

1993, [48]
Szilagyi, 1996, [60]
Chen et al, 1997, [11]

Background

Wavelets in Meteorology and Oceanography

- Meyers et al [20].
- Szilagyi et al[21].
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Szilagyi, 1996, [60]
Chen et al, 1997, [11]

Background

Wavelets in Geology and Geophysics

- Complex seismic trace analysis[23].

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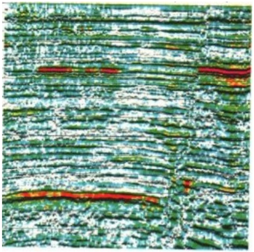
[23] Taner, 1979, [61]

Background

Wavelets in Geology and Geophysics



Reflection strenght

Instantaneous frequency

Apparent polarity

Background

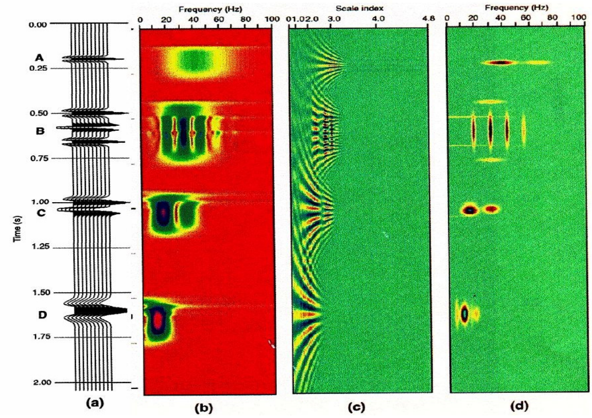
Wavelets in Geology and Geophysics

- Complex seismic trace analysis.
- STFT, CWT, MPD [24].

Chakraborty and Okaya, 1995, [9]

Background

Wavelets in Geology and Geophysics



Background

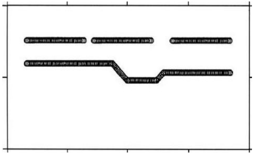
Wavelets in Geology and Geophysics

- Complex seismic trace analysis.
- STFT, CWT, MPD.
- Acoustic wavelet transform[25].

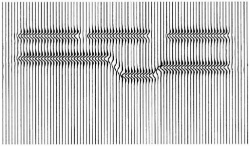
[25] Wu et al, 1998, [66]

Background

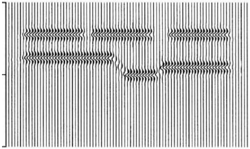
Wavelets in Geology and Geophysics



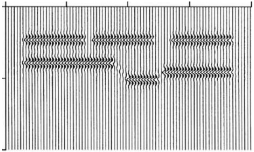
Synthetic model



Daubechies-4



Daubechies-8



Daubechies-12

Background

Wavelets in Geology and Geophysics

- Complex seismic trace analysis.
- STFT, CWT, MPD.
- Acoustic wavelet transform.
- Fast Kirchhoff migration[26].

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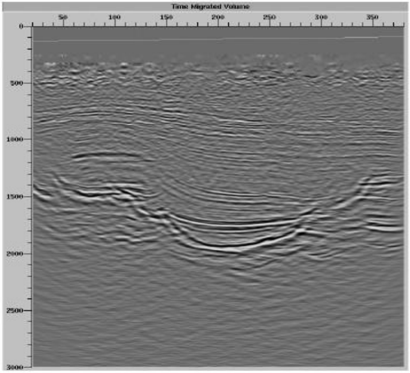
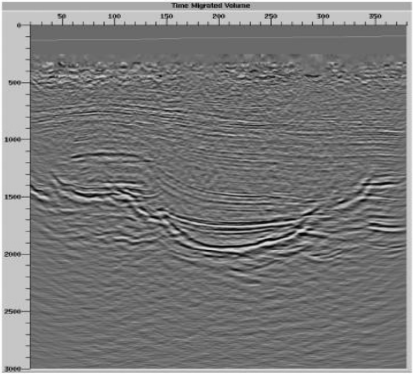
[26] Zheludev, 2002, [69]

Background

Wavelets in Geology and Geophysics



Conventional prestack migration



Wavelet prestack migration

Background

Wavelets in Geology and Geophysics

- Prestack multiscale Kirchhoff migration[27].

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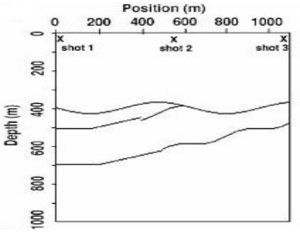
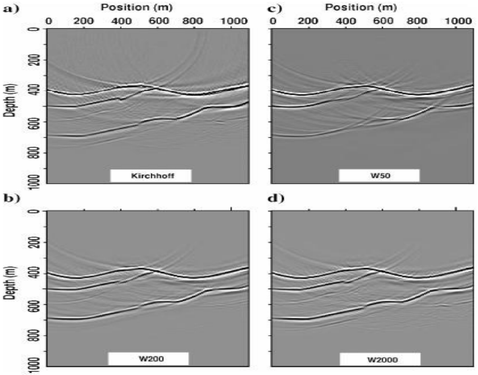
[27] Yu et al, 2004, [67]

Background

Wavelets in Geology and Geophysics



Reflectivity model



Migrated image

Background

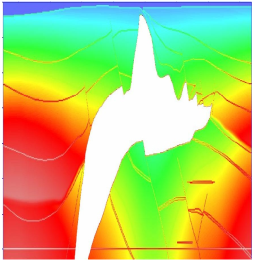
Wavelets in Geology and Geophysics

- Prestack multiscale Kirchhoff migration.
- Beamlet prestack depth migration[28].

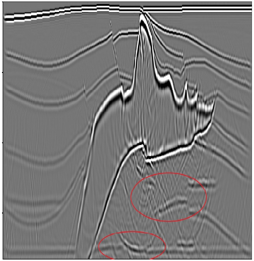
Chen et al, 2004, [12]

Background

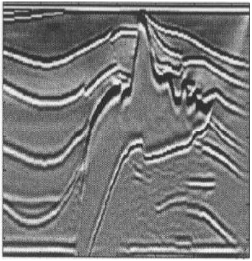
Wavelets in Geology and Geophysics



SEG-EAGE salt model



Split step migration



Beamlet prestack depth migration

Background

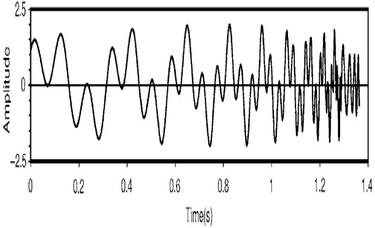
Wavelets in Geology and Geophysics

- Prestack multiscale Kirchhoff migration.
- Beamlet prestack depth migration.
- Time frequency continuous wavelet transform TFCWT[29].

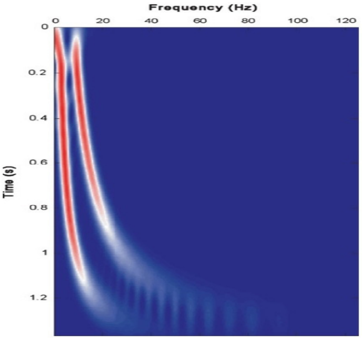
Sinha et al, 2005, [55]

Background

Wavelets in Geology and Geophysics



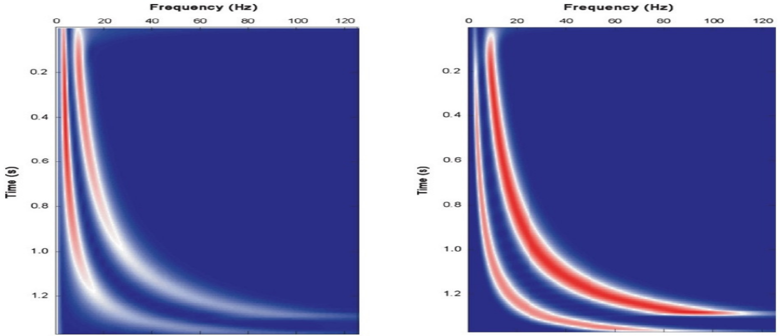
Chirp signal



SFFT spectrum

Background

Wavelets in Geology and Geophysics



CWT spectrum TFCWT spectrum

Background

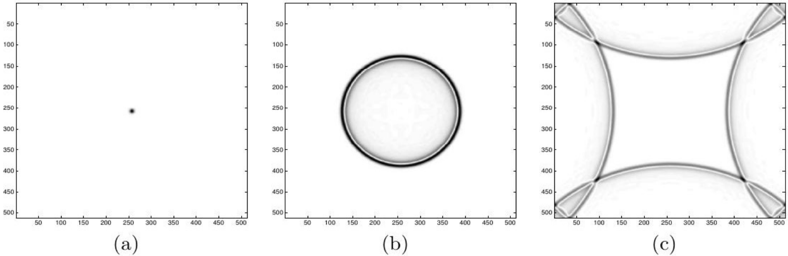
Wavelets in Geology and Geophysics

- Fast discrete curvelet transform[30].

Candes et al, 2006,[8]

Background

Wavelets in Geology and Geophysics



a) Delta function b) Approximate solution at $t = 0.25s$ c) Approximate solution at $t = 0.75s$

Image size	T_{Fwd}(s)	T_{Adj}(s)	T_{Inv}(s)	T_{Fwd}/T_{FFT}	ℓ^2 error
128×128	0.088832	0.091578	1.006522	24.6756	1.4430e-06
256×256	0.376838	0.390533	4.002353	19.0322	8.8154e-07
512×512	2.487052	2.579102	35.09599	18.2202	5.3195e-07
1024×1024	16.47702	16.87764	129.3631	28.9579	3.2390e-07
2048×2048	62.42980	65.09365	566.1732	24.1920	3.4305e-06

Background

Wavelets in Geology and Geophysics

- Fast discrete curvelet transform.
- Fast discrete curvelet transform - reconstruct seismic data[31].

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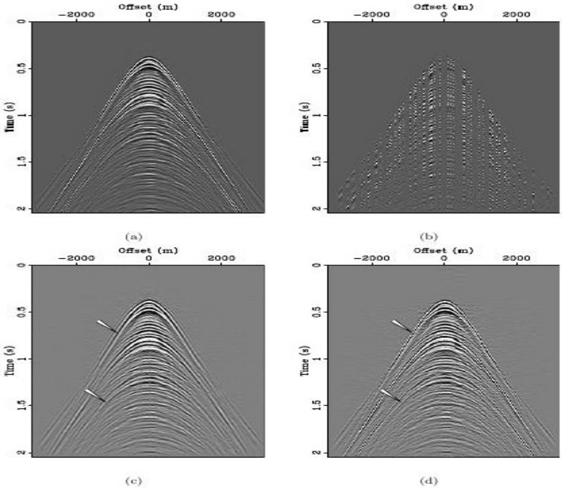
[31] Herrmann, 2007, [42]

Background



a) Fully sampled real data shot gather b) Ramdomly subsampled shot gather with 80% of the traces missing in the receiver and shot directions c) Curvelet-based recovery d) Curvelet-based recovery with focusing

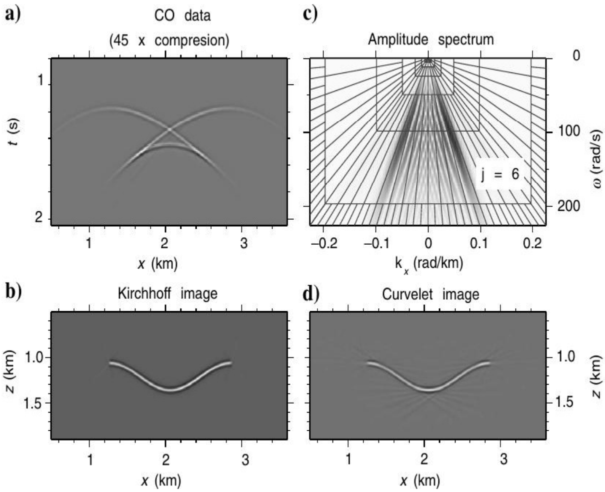
Background

Wavelets in Geology and Geophysics

- Fast discrete curvelet transform.
- Fast discrete curvelet transform - reconstruct seismic data.
- Zero offset Kirchhoff migration with curvelets[32].

[32] Douma and De Hoop, 2007, [20]

Background


a)
CO data
(45 x compresion)
t (s)
x (km)
c)
Amplitude spectrum
j = 6
k_x (rad/km)
ω (rad/s)
b)
Kirchhoff image
z (km)
x (km)
d)
Curvelet image
z (km)
x (km)

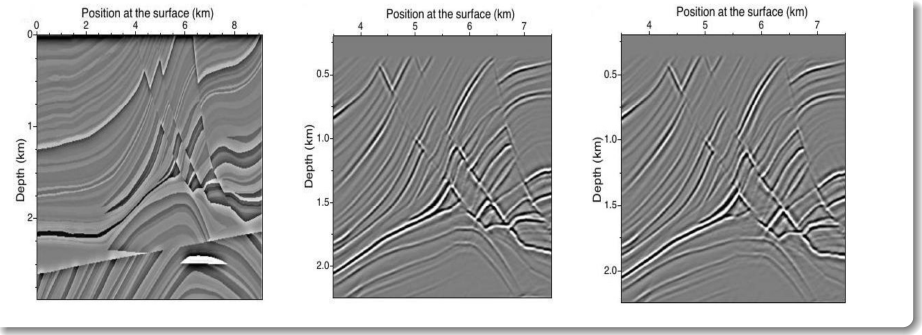
Background

Wavelets in Geology and Geophysics

- Fast discrete curvelet transform.
- Fast discrete curvelet transform - reconstruct seismic data.
- Zero offset Kirchhoff migration with curvelets.
- Seismic demigration/migration with curvelets[33].

[33] Chauris and Nguyen, 20098,[10]

Background


Position at the surface (km)
0 2 4 6 8
Depth (km)
0 1 2
Position at the surface (km)
4 5 6 7
Depth (km)
0.5 1.0 1.5 2.0
Position at the surface (km)
4 5 6 7
Depth (km)
0.5 1.0 1.5 2.0

Current works

- Generation of synthetic seismic data using seismic unix.
- Migration of synthetic seismic data using seismix unix.

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- Forward modeling of acoustic wave equation using the pseudospectral method (second order in time, second, fourth and sixth order in space).

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- Forward modeling of acoustic wave equation using the pseudospectral method (second order in time, second, fourth and sixth order in space).

Reverse Time Migration

- Forward propagation
- Backward propagation
- Condition image (cross-correlation)

$$I_{cc}(\mathbf{x}) = \int P_F(\mathbf{x}, t) P_B(\mathbf{x}, t) dt$$

Reverse Time Migration

Acoustic wave equation

$$\frac{1}{c^2}\frac{\partial^2 u(\mathbf{x},t)}{\partial t^2} - \nabla^2 u(\mathbf{x},t) = s(\mathbf{x},t) \qquad (1)$$

$u(\mathbf{x},t)$: Wavefield at time t
$\mathbf{x} = (x,y,z)$: Position vector
$c = c(\mathbf{x})$: Acoustic propagation velocity
$s(\mathbf{x},t)$: Source term
$\nabla^2 = \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}\right)$: The Laplacian operator in Cartesian coordinates

Reverse Time Migration

Finite Difference

Forward propagation (Second order in time and space)

$$U_{i,j}^{n+1} = 2U_{i,j}^{n} - U_{i,j}^{n-1} + \nu^2[U_{i+1,j}^{n} + U_{i-1,j}^{n} + U_{i,j+1}^{n} + U_{i,j-1}^{n} - 4U_{i,j}^{n}] + S_{i,j}^{n} \quad (2)$$

with

$$\nu = \frac{c_{i,j}\Delta t}{h} \quad (3)$$

Backward propagation (Second order in time and space)

$$\widetilde{U}_{i,j}^{n+1} = 2\widetilde{U}_{i,j}^{n} - \widetilde{U}_{i,j}^{n-1} + \nu^2[\widetilde{U}_{i+1,j}^{n} + \widetilde{U}_{i-1,j}^{n} + \widetilde{U}_{i,j+1}^{n} + U_{i,j-1}^{n} - 4\widetilde{U}_{i,j}^{n}] + \widetilde{S}_{i,j}^{n} \quad (4)$$

Reverse Time Migration

Pseudospectral method

Rewriting the wave equation

$$\frac{\partial^2 u(\mathbf{x},t)}{\partial t^2} = -L^2 u(\mathbf{x},t) \tag{5}$$

with

$$-L^2 = c^2(\mathbf{x})\nabla^2$$

The formal solution of the equation 5 with initial conditions $\frac{\partial u(\mathbf{x},t)}{\partial t}(t = 0) = \dot{u}_0$ and $u(\mathbf{x}, t = 0) = u_0$ is given by

$$u(\mathbf{x},t) = \cos(Lt)u_0 + L^{-1}\sin(Lt)\dot{u}_0 \tag{6}$$

Reverse Time Migration

The wavefields $u(\mathbf{x}, t + \Delta t)$ and $u(\mathbf{x}, t - \Delta t)$ can be evaluated by equation 6. Adding these two wavefields result is

$$u(\mathbf{x}, t + \Delta t) + u(\mathbf{x}, t - \Delta t) = 2\cos(L\Delta t)u(\mathbf{x}, t) \tag{7}$$

If we take for $\cos(L\Delta t)$ its second-order $(1 - \frac{(L\Delta t)^2}{2})$ Taylor-series expansion, we obtain

$$u(\mathbf{x}, t + \Delta t) - 2u(\mathbf{x}, t) + u(\mathbf{x}, t - \Delta t) = -\Delta t^2 L^2 u(\mathbf{x}, t) \tag{8}$$

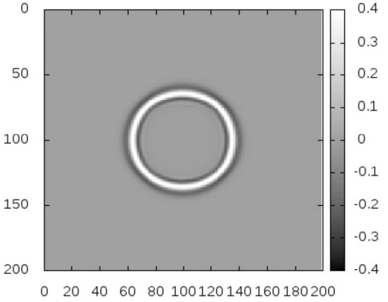
Reverse Time Migration

Using a pseudospectral method (Etgen, 1986 [24], Zhang et al., 2007 [68]) for the spatial derivatives, we can express equation 8 as:

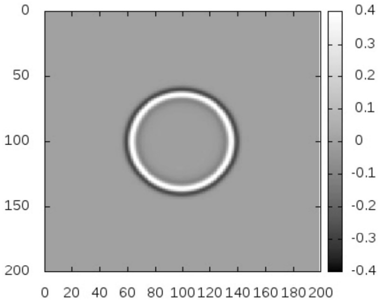
$$u(\mathbf{x}, t + \Delta t) - 2u(\mathbf{x}, t) + u(\mathbf{x}, t - \Delta t) = \Delta t^2 [c^2(\mathbf{x}) FT^{-1} (k_x^2 + k_y^2 + k_z^2) FT] u(\mathbf{x}, t)$$

Reverse Time Migration

Snapshot at 6 s



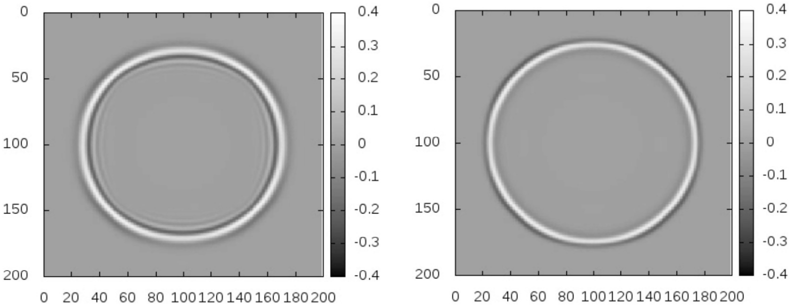
Finite difference



Pseudoespectral

Reverse Time Migration

Snapshot at 12 s



Finite difference Pseudoespectral

Current works

Finite difference method
https://youtu.be/rhCRqaEHXqA

Pseudoespectral method
https://youtu.be/5MOmgKzpk

Current works

- Discrete approximation of a function using the Haar system.

Current works

Haar system

$$f(x) = \sum_{k=0}^{2^J-1} a_{J,k} p_{J,k}(x) + \sum_{j=J}^{\infty} \sum_{k=0}^{2^j-1} b_{j,k} h_{j,k}(x) \tag{9}$$

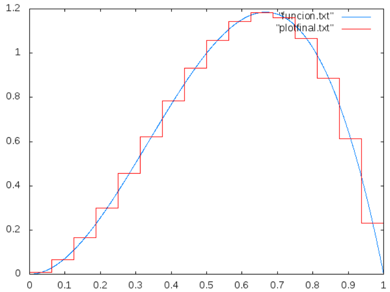
where

$$a_{J,k} = \langle f, p_{J,k} \rangle$$

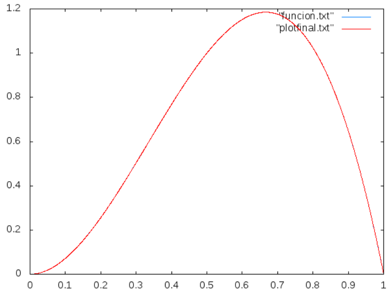
$$b_{j,k} = \langle f, h_{j,k} \rangle$$

Current works

Haar system



$f(x) = 8x(1-x); J = 0, j = 3$



$f(x) = 8x(1-x); J = 0, j = 15$

Current works

- Discrete approximation of a function using the Haar system.
- C language implementation for Phase shift migration.

Phase shift Migration

2D acoustic wave equation

$$\frac{\partial^2 P}{\partial z^2} + \frac{\partial^2 P}{\partial x^2} - \frac{1}{c^2}\frac{\partial^2 P}{\partial t^2} = 0 \tag{10}$$

Applying the Fourier transform to the equation 10 we have

$$\frac{\partial^2 P}{\partial z^2} + \left(\frac{\omega^2}{c^2} - k_x^2\right) P = 0 \tag{11}$$

Let $A^2 = \frac{\omega^2}{c^2} - k_x^2$

Phase shift Migration

Then

$$\left(\frac{\partial^2}{\partial z^2} + A^2\right) P = 0 \tag{12}$$

$$\left(\frac{\partial}{\partial z} + iA\right)\left(\frac{\partial}{\partial z} - iA\right) P = 0 \tag{13}$$

If $c = c(z)$ but each subinterval $[z_i, z_{i+1}]$ is to be $c(z) = constant$, then we can solve the equation 13.

Then

$$\frac{\partial}{\partial z} P(k_x, z, \omega) = -i\sqrt{\frac{\omega^2}{c^2} - k_x^2} P(k_x, z, \omega) \tag{14}$$

Phase shift Migration

The equation to extrapolate the wavefield is

$$P(\omega, k_x, z + \Delta z) = P(\omega, k_x, z)e^{-ik_z \Delta z} \tag{15}$$

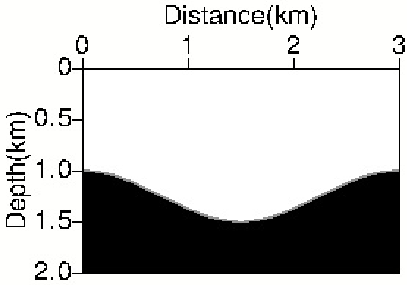
where

$$k_z = \sqrt{\frac{\omega^2}{c^2} - k_x^2}$$

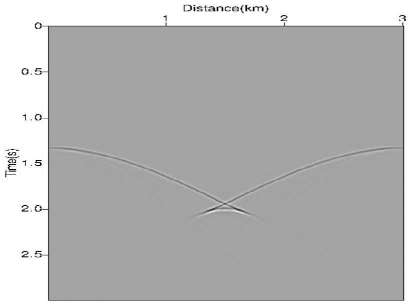
The migrated section on each $z + \Delta z$ level is given by (image condition at $t = 0$)

$$P(x, z + \Delta z) = \int d\omega \int P(\omega, k_x, z)e^{-ik_z \Delta z} e^{-ik_x x} dk_x \tag{16}$$

Phase shift Migration

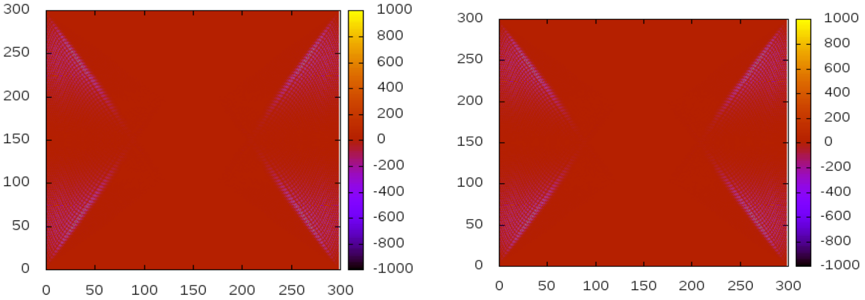


Sinclina model



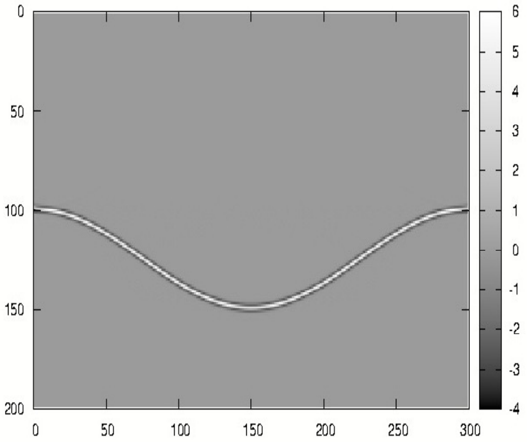
Seismic section

Phase shift Migration



Real part of the spectrum   Imaginary part of the spectrum

Phase shift Migration



Current works

- Discrete approximation of a function using the Haar system.
- C language implementation for Phase shift migration.
- Analyze and study the possibility of implementing the phase shift migration using the discrete wavelet transform with the system haar

Work perspectives

- Is it possible to calculate the Laplacian (2D, 3D) using a different method that does not use a finite differences scheme or pseudo-espectral?
- Is it possible to improve the dispersion in wave propagation using a different method to calculate the Laplacian (2D, 3D), for example, a wavelet transform or another transform?
- Is it feasible to find other 2D, 3D orthogonal transformation that can be used in another method of migration, such as reverse time migration, allowing improve the images obtained through the analysis of the signals recorded on the surface?
- Search for a methodology to find the subsurface velocity field (2D, 3D) through the analysis of the signals recorded on the surface.

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