Dealing with Feasibility and Solution Quality in a Rebalancing Vehicle Routing Problem

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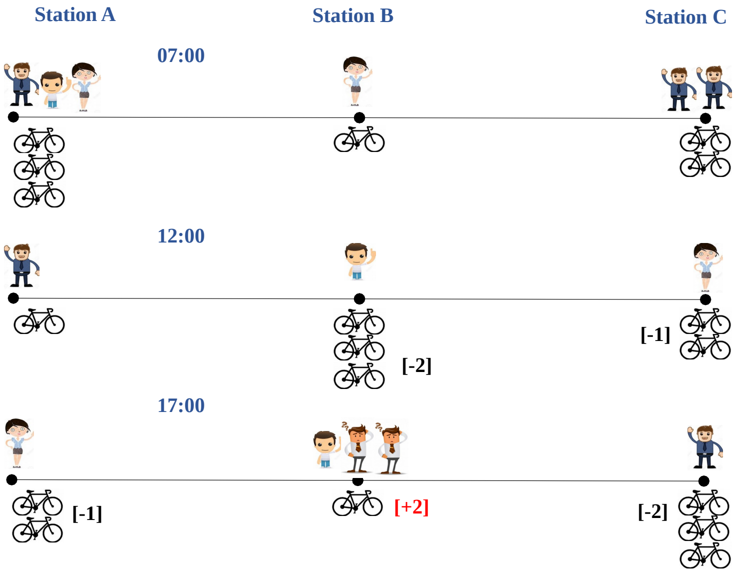
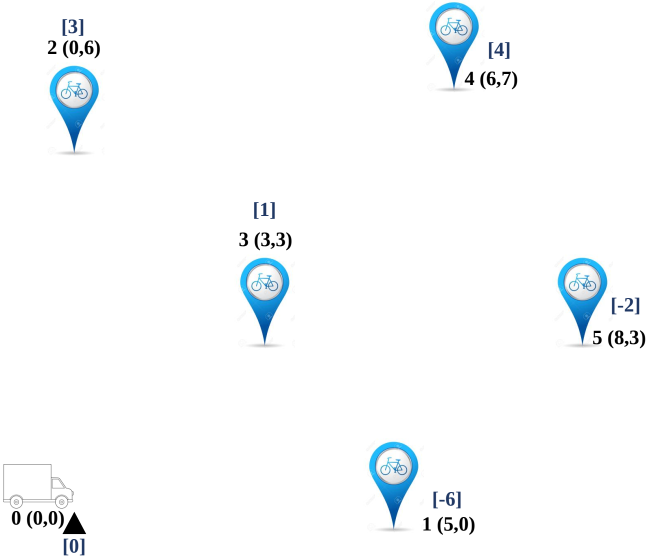
Outline

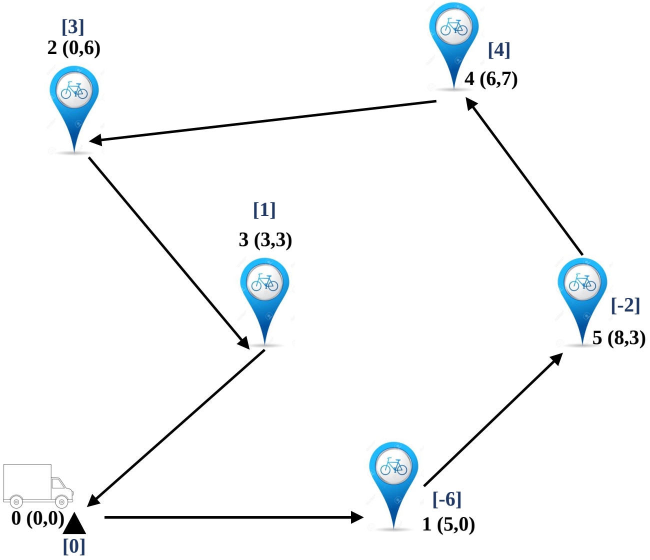
Balancing a BSS



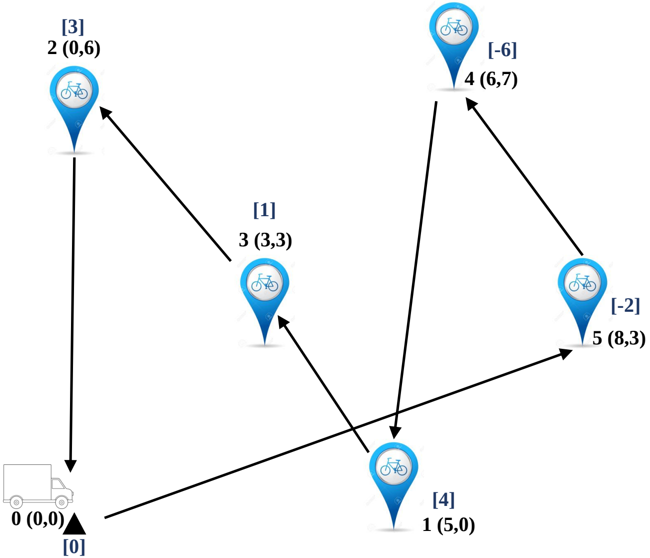
Pick up and Delivery TSP



Pick up and Delivery TSP



Pick up and Delivery TSP



Solution Strategies – Single Vehicle Case

- Mixed Integer Programming Models (MILPs)
  - Traveling Salesman Problem (TSP)
  - Pick up and Delivery TSP (PDTSP)
  - PDTSP with Split Demand (PDTSPSD)
- Heuristic algorithms
  - Nearest Neighbor (TSP)
  - Extensions of Nearest Neighbor for PDTSP and PDTSPSD
- Metaheuristic algorithms
  - Greedy Randomized Adaptive Search Procedure (GRASP)
  - Path Relinking
  - Variable Neighborhood Descent (VND)
- Hybrid approaches (matheuristics)
  - MILP based local search operators
  - MILP based post-optimization procedures
  - PDTPSP decomposition

A GRASP Algorithm

Algorithm 1 GRASP

$f^* \leftarrow \infty$
for $i = 1$ to *GRASPIterations* **do**
  $S \leftarrow$GreedyRandomAlgorithm()
  $S \leftarrow$LocalSearch(S)
  if $f(S) < f^*$ **then**
    $S^* \leftarrow S$
    $f^* \leftarrow f(S)$
  end if
end for
return S^*

Our GRASP Algorithm

Algorithm 2 GRASP + VND

$f^* \leftarrow \infty$
for $i = 1$ to *GRASPIterations* **do**
  $S \leftarrow$GreedyRandomAlgorithm()
  $S \leftarrow$VND(S)
  if $f(S) < f^*$ **then**
    $S^* \leftarrow S$
    $f^* \leftarrow f(S)$
  end if
end for
return S^*

VND Structure

Seven neighborhoods within a VND method

- Destroy and Repair
- 2-opt
- 3-opt
- Or-opt(λ), $\lambda = 2, 3$
- Forward insertion
- Backward insertion

Our GRASP Algorithm

Algorithm 3 GRASP + VND

1: $f^* \leftarrow \infty$
2: **for** $i = 1$ to *GRASPIterations* **do**
3: $S \leftarrow$GreedyRandomAlgorithm()
4: $S \leftarrow$VND(S)
5: **if** $f(S) < f^*$ **then**
6: $S^* \leftarrow S$
7: $f^* \leftarrow f(S)$
8: **end if**
9: **end for**
10: **return** S^*

Our GRASP Algorithm

Algorithm 4 Greedy Random Algorithm

$S \leftarrow \emptyset$
Evaluate the incremental costs of the candidate elements
while S is not a complete solution **do**
    Build the restricted candidate list (RCL)
    Select an element s from the RCL at random
    $S \leftarrow S \cup \{s\}$
    Reevaluate the incremental costs
end while
return S

Our GRASP Algorithm

Example of an unfeasible constructive solution

- Vehicle capacity (Q)=10

Station		1	2	3	4	5	6	7	8	9	10	11	12	13	14
Demand		4	6	-3	-3	-3	4	3	-7	0	9	-7	10	-6	-7
Load	10	6	0	3	6	9	5	2	9	9	0	7	-3	3	10

Example of an infeasible constructive solution

- Vehicle capacity (Q)=10

Station		1	2	3	4	5	6	7	8	9	10	11	12	13	14
Demand		4	6	-3	-3	-3	4	3	-7	0	9	-7	10	-6	-7
Load	10	6	0	3	6	9	5	2	9	9	0	7	-3	3	10

- Set of fixed ordered stations : Stations from 1 to 11
- Set of stations to remove and insert in a different position: Stations from 12 to 14

Repairing an Unfeasible Solution – MILP

- Sets
  - $\mathcal{N}$: Set of stations
  - $\mathcal{N}_s$: Set of fixed ordered stations
  - $\mathcal{N}_r$: Set of stations to insert
  - $\mathcal{A}$: Set of arcs
- Parameters
  - c_{ij} : Cost of traveling from station i to station j
  - q_i : Demand or slack of bicycles in station i
  - Q : Vehicle capacity
- Decision Variables
  - s: Surplus of bikes in the solution
  - $y_{ij} = \begin{cases} 1 & \text{if arc } (i,j) \text{ is used in the solution} \\ 0 & \text{otherwise} \end{cases}$
  - $x_{ij} = \begin{cases} 1 & \text{if station } i \text{ is visited anytime before station } j \\ 0 & \text{otherwise} \end{cases}$
  - l_i : Load of the vehicle after visiting i^{th} station
  - l^+, l^- : Maximum and minimum load in the solution

Repairing an Unfeasible Solution – MILP

$$\min f = s \quad (1)$$

subject to,

$$x_{0i} = 1 \qquad \forall\ i \in \mathcal{N} \backslash \{0\} \quad (2)$$

$$x_{ij} + x_{ji} = 1 \qquad \forall\ (i,j) \in \mathcal{A} \quad (3)$$

$$y_{ij} \leq x_{ij} \qquad \forall\ (i,j) \in \mathcal{A} \quad (4)$$

$$x_{ij} = 1 \qquad \forall\ i \in \mathcal{N}_f, j \in \mathcal{N}_f, j > i \quad (5)$$

$$\sum_{j \in \mathcal{N}} y_{ij} = 1 \qquad \forall\ i \in \mathcal{N} \quad (6)$$

$$y_{i,i+1} + \sum_{j \in \mathcal{N}_r} y_{ij} = 1 \qquad \forall\ i \in \mathcal{N}_f \quad (7)$$

Repairing an Unfeasible Solution – MILP

$$\sum_{j \in \mathcal{N}} y_{ij} - \sum_{j \in \mathcal{N}} y_{ji} = 0 \qquad \forall\ i \in \mathcal{N} \qquad (8)$$

$$l_j \geq l_i - q_j - 2Q(1 - y_{ij}) \qquad \forall\ (i,j) \in \mathcal{A} \qquad (9)$$

$$l^+ \geq l_i \qquad \forall\ i \in \mathcal{N} \qquad (10)$$

$$l^- \leq l_i \qquad \forall\ i \in \mathcal{N} \qquad (11)$$

$$l^+ - l^- = Q + s \qquad (12)$$

$$y_{ij} \in \{0, 1\} \qquad \forall\ i \in \mathcal{N}, j \in \mathcal{N} \qquad (13)$$

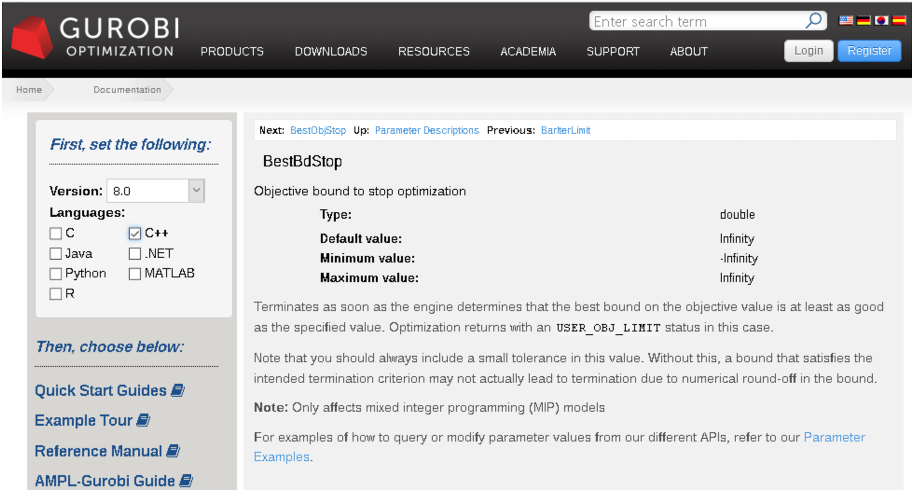
$$x_{ij} \in \{0, 1\} \qquad \forall\ i \in \mathcal{N}, j \in \mathcal{N} \qquad (14)$$

$$l_i \geq 0 \qquad \forall\ i \in \mathcal{N} \qquad (15)$$

$$l^+, l^- \geq 0 \qquad (16)$$

Speeding up MILP Performance

- Time limit
- Objective bound to stop optimization



An Hybrid Constructive Phase

Algorithm 5 GRASP + VND

$c \leftarrow \infty$
$f^* \leftarrow \infty$
for $i = 1$ to *GRASPIterations* **do**
    $S \leftarrow$GreedyRandomAlgorithm(c)
    $S \leftarrow$VND(S)
    if $f(S) < f^*$ **then**
        $S^* \leftarrow S$
        $f^* \leftarrow f(S)$
    end if
end for
return S^*

An Hybrid Constructive Phase

Algorithm 6 Greedy Random Algorithm

$S \leftarrow$GreedyRandomAlgorithm()
while S is unfeasible **do**
    if $f(S) < c$ **then**
        Solve $RepairMILP(S)$
    else
        $S \leftarrow$GreedyRandomAlgorithm()
    end if
end while
if $c > f(S)$ **then**
    $c = f(S)$
end if
return S^*, c

- Dataset
  - Instances taken from: `https://hhperez.webs.ull.es/PDsite/index.html`
  - Instances with 20, 30, 50, 100 nodes were tested
- Software
  - All the algorithms were implemented on C++ (Visual Studio)
  - Mathematical models were solved using Gurobi Optimizer 7.5
- Computer features
  - Intel core i7, 8Gb RAM.
  - OS: Windows 10 (x86-64)

Preliminary Results – MILP-based Constructive Algorithm

	Original constructive algorithm					Repair constructive algorithm - MILP					
	Best		Average	Average	Average %	Best		Average	Average	Average	Average %
$\mathcal{N}$	z	Load	z	Load	Feasible Sols	z	Load	z	Load	Lower Bound	Feasible Sols
20	7853	10	9811.36	12.24	37.00%	7647	10	9969.73	10.21	0.18	85.00%
30	12387	10	14019.04	15.08	5.50%	11396	10	14131.74	11.46	1.03	34.00%
50	18345	10	21346.69	12.66	28.00%	17618	10	21591.77	10.56	0.35	69.00%
100	32200	10	35516.16	15.48	10.50%	32203	10	35915.57	12.33	0.61	26.50%

Preliminary Results – Hybrid Constructive Algorithm

	Original constructive algorithm					Hybrid constructive algorithm				
$\mathcal{N}$	Best Distance	Load	Average Distance	Average Load	Average % Feasible Sols	Best Distance	Load	Average Distance	Average Load	Average % Feasible Sols
20	7610	10	9848.63	12.05	36.50%	7610	10	9863.77	10	100%
30	12122	10	14089.67	14.58	10.00%	11707	10	13973.72	10	100%
50	18590	10	21526.07	13.06	27.50%	17691	10	21438.71	10	100%
100	31743	10	35637.01	15.79	13.50%	30889	10	35605.6	10	100%

Current and Future Work

- Improve the performance of destroy strategy finding a different criterion to remove stations.
- Solve larger instances of the PDTSP via GRASP+VND and compare our results with reported benchmarks.
- Extend our solutions algorithms to the multiple vehicle case.

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